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Envelope solitons induced by high-order effects of light-plasma interaction

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Abstract. The nonlinear coupling between light beams and non-resonant ion density perturbations in a plasma is considered, taking into account the relativistic particle mass increase and the light beam ponderomotive force. A pair of equations comprising a nonlinear Schrödinger equation for light beams and a driven (by the light beam pressure) ion-acoustic wave response is derived. It is shown that the stationary solutions of our nonlinear equations can be represented in the form of a bright and dark/gray soliton for the one-dimensional problem. We also present numerical results which exhibit that our bright soliton solutions are stable exclusively for the values of the parameters compatible with our theory.

PACS. 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.) – 52.40.Db Electromagnetic (nonlaser) radiation interactions with plasma – 52.35.Sb Solitons; BGK modes

1 Introduction

Recently, investigations concerning the nonlinear dynamics governed by a multi-dimensional cubic-quintic nonlinear Schrödinger equation (NLSE) have received a great deal of attention. In this context, both localized vortex solitons and non-localized optical vortices have been studied [1]. The cubic-quintic $(2+1)$-dimensional NLSE has been used to study the stability of spinning ring solitons [2] and theoretical investigations to find solitary solutions for the cubic-quintic $(1+1)$-dimensional NLSE have been carried out. Dark solitary waves in the limit of small amplitudes have been found, where the NLSE was reduced to a Kortweg-de Vries equation (KdVE) [3]. Moreover, both algebraic solitary wave solutions [4] and traveling-wave solutions [5] have been found and criteria for the existence and stability of soliton solutions have been established [6]. Additionally, a theory which connects envelope solitons of a wide class of generalized NLSEs with solitons of a wide class of generalized KdVE have been recently carried out for arbitrary amplitudes [7]; in particular, the theory was applied to find analytical bright, gray and dark envelope soliton solutions of the cubic-quintic NLSE and some other types of nonlinearities [7–9].

It is well known that nonlinear interactions between intense laser beams and a plasma are responsible for numerous nonlinear phenomena including parametric instabilities [10], density cavitation, self-focusing and filamentation of light [11–13], as well as the generation of large amplitude electric fields to be used to accelerate charged-particle bunches [14]. Intense laser beams can cause density modifications through the ponderomotive force, enhance the electron mass due to relativistic effects and produce electron Joule heating. Both relativistic electron-mass variation and pump wave effects have been suitably considered for describing the beam self-focusing [15]. Furthermore, a Hamiltonian approach to describe the dynamics of solitary waves in the Zakharov model equations has been devoted [16]. The interplay between the ponderomotive, relativistic and Joule heating non–linearities has been examined [11] in the context of laser plasma experiments and also in ionospheric modifications of the Earth's ionosphere by powerful radar beams.

In this paper, we investigate nonlinear interactions between circularly polarized light beams and non-resonant density perturbations in a uniform unmagnetized plasma, taking into account the combined effects of the light pressure induced ion density fluctuations and increased electron mass. We neglect low-frequency (quasi-static) magnetic fields [17–20] that are induced by strong plasma density and temperature inhomogeneities [17,19], inverse Faraday effect [18], and photon spin [20]. Spontaneously excited megagauss magnetic fields would not affect the

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light wave propagation as long as the light wave frequency is much larger than the electron gyrofrequency. Under suitable physical conditions for which our system can be described by a (1 + 1)-dimensional cubic-quintic NLSE for the complex electromagnetic field amplitude, we analytically obtain bright, gray and dark envelope solitons. Finally, a stability analysis has been carried out, which shows that our bright soliton solutions are stable.

2 Basic equations

We consider the propagation of a large amplitude circularly polarized electromagnetic wave with an electric field $\mathbf{E} = E(\hat{\mathbf{x}} + \mathrm{i}\hat{\mathbf{y}})\exp(-\mathrm{i}\omega t + \mathrm{i}\mathbf{k}\cdot\mathbf{R})$, where ω is the wave frequency and $\mathbf{k}$ is the wavevector. The light equation in the presence of electron density perturbations in a plasma is obtained from

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\partial_t\mathbf{E}, \tag{1}$$

with

$$\mathbf{J} = -e(n_0 + n_1)\mathbf{v}_e, \tag{2}$$

$$\mathbf{B} = \nabla\times\mathbf{A}, \tag{3}$$

$$\mathbf{E} = -\frac{1}{c}\partial_t\mathbf{A}, \tag{4}$$

and

$$\partial_t\mathbf{p}_e = -e\mathbf{E}, \tag{5}$$

where $\mathbf{B}$ is the wave magnetic field, $\mathbf{A}$ is the vector potential, n_0 and n_1 are the unperturbed and perturbed electron number densities, $\mathbf{v}_e$ is the particle quiver velocity induced by the photons, $\mathbf{p}_e = m_e\mathbf{v}_e$ is the momentum, $m_e = m_0/(1-\mathbf{v}_e^2/c^2)^{1/2}$ is the mass, m_0 is the rest electron mass, e is the magnitude of the electron charge, and c is the speed of light in vacuum. The perturbation of the number density n_1 is reinforced by the light ponderomotive force. The nonlinear high-frequency current density $-en_1\mathbf{v}_e$ in equation (2) arises due to the beating of the slow density perturbations and the electron quiver velocity in the light wave vector potential. For our purposes, we have

$$\mathbf{v}_e = \frac{e}{m_0 c}\frac{\mathbf{A}}{\gamma_e}, \tag{6}$$

in view of equations (4, 5). Here we have denoted $\gamma_e = \sqrt{1 + e^2\mathbf{A}^2/m_0^2c^4}$. The nonlinear term in γ_e arises due to the electron mass increase in the light wave fields.

Combining equations (1, 2, 3) and (6) we obtain

$$\partial_t^2\mathbf{A} - c^2\nabla^2\mathbf{A} + \omega_p^2\,(1+N)\,\frac{\mathbf{A}}{\gamma_e} = 0, \tag{7}$$

where $\omega_p = (4\pi n_0 e^2/m_0)^{1/2}$ is the plasma frequency, $N = n_1/n_0$, and where we have introduced the Coulomb gauge $\nabla\mathbf{A} = 0$.

Supposing that $\mathbf{A} = \mathbf{A}_s(\mathbf{r},\tau)\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{R} - \mathrm{i}\omega t)$ + complex conjugate, where $\mathbf{r}$ and τ represent slowly varying space and time coordinates, we obtain from equation (7)

$$2\mathrm{i}\omega(\partial_\tau + \mathbf{v}_g\cdot\nabla_{\mathbf{r}})\mathbf{A}_s + c^2\nabla_{\mathbf{r}}^2\mathbf{A}_s + \Omega^2\mathbf{A}_s - \frac{\omega_p^2(1+N)\mathbf{A}_s}{\gamma_e} = 0, \tag{8}$$

where $\mathbf{v}_g = \mathbf{k}c^2/\omega$ is the group velocity of the light wave, and $|\partial_\tau\mathbf{A}| \ll \omega\mathbf{A}$ has been invoked in view of the WKB approximation. We have denoted $\Omega^2 = \omega^2 - c^2k^2$.

We now derive the equation for low-phase velocity (in comparison with the electron thermal speed) density perturbations that are driven by the light wave ponderomotive force. The governing equations are the inertialess electron momentum equation

$$0 = e\nabla_{\mathbf{r}}\phi - m_0c^2\nabla_{\mathbf{r}}\gamma_e - T_e\nabla_{\mathbf{r}}\ln\,(n_e/n_0), \tag{9}$$

the ion continuity equation

$$\partial_\tau n_i + \nabla_{\mathbf{r}}\cdot(n_i\mathbf{u}_i) = 0, \tag{10}$$

and

$$\partial_\tau\mathbf{u}_i + (\mathbf{u}_i\cdot\nabla_{\mathbf{r}})\mathbf{u}_i = -\frac{e}{m_i}\nabla_{\mathbf{r}}\phi - \frac{T_i}{m_i}\nabla_{\mathbf{r}}\ln\,(n_i/n_0), \tag{11}$$

where ϕ is the electrostatic potential, $\mathbf{u}_i$ is the fluid velocity associated with the plasma slow motion, and $T_e(T_i)$ is the electron (ion) temperature. The second term in the right-hand side of (9) represents the light pressure. Equations (9) to (11) form a closed system when the quasi-neutrality $n_e = n_i$ is invoked. The light ponderomotive force acting on the ion fluid is insignificant. Equation (9) shows that the electrons are pushed away from the region of maximum light intensity, and reinforce a space charge electric field $(-\nabla\phi)$ and the associated density fluctuations. The light ponderomotive force is transmitted to ions through the space charge electric field.

Adding equations (9, 11) and letting $n_{e,i} = n_0 + n_1$, $\mathbf{u}_i = \mathbf{u}_{i0} + \mathbf{u}_{i1} = \mathbf{u}_{i1}$, we obtain for $n_1 \ll n_0$ and $|\left(\mathbf{u}_{i1}\cdot\nabla_{\mathbf{r}}\right)\mathbf{u}_{i1}| \ll |\partial_\tau\mathbf{u}_{i1}|$

$$\partial_\tau\mathbf{u}_{i1} = -\frac{m_0}{m_i}c^2\nabla_{\mathbf{r}}\gamma_{e1} - \frac{C_s^2}{n_0}\nabla_{\mathbf{r}}n_1, \tag{12}$$

where for consistency we have assumed $e^2\mathbf{A}_s^2/m_0^2c^4 \ll 1$ and, consequently, introduced a small perturbation γ_{e1} of the electron relativistic factor γ_e ($\gamma_e \approx 1 + \gamma_{e1}$, where $\gamma_{e1} = e^2\mathbf{A}_s^2/2m_0^2c^4$) and $C_s = [(T_e + T_i)/m_i]^{1/2}$ is the effective sound speed. Combining equation (12) with the linearized version of equation (10), we obtain

$$\partial_\tau^2 N - C_s^2\nabla_{\mathbf{r}}^2 N = \frac{m_0}{m_i}c^2\nabla_{\mathbf{r}}^2\gamma_{e1}. \tag{13}$$

In the small amplitude limit, *viz.* $\gamma_e \ll 1$, equation (8) becomes

$$2\mathrm{i}\omega(\partial_\tau + \mathbf{v}_g\cdot\nabla_{\mathbf{r}})\mathbf{A}_s + c^2\nabla_{\mathbf{r}}^2\mathbf{A}_s + \left(\Omega^2 - \omega_p^2\right)\mathbf{A}_s - \omega_p^2\left[(N - \gamma_{e1}) - N\gamma_{e1}\right]\mathbf{A}_s = 0. \tag{14}$$

Equations (13, 14) are the desired equations for coherent light beams that are coupled with non-resonant density perturbations in an electron-ion plasma. Note that, in principle, the quantities $(N-\gamma_{e1})$ and $N\gamma_{e1}$, involved in equation (14), could be of the same order. In the following, this physical circumstance will be considered and, to this end, we seek possible stationary nonlinear solutions of equations (13, 14) in the form of envelope solitons.

3 Envelope solitons

We introduce $\xi = \mathbf{r} - \mathbf{V}\tau$, where $\mathbf{V}$ is the velocity of the nonlinear waves, and assume $\mathbf{A}_s = \mathbf{a}(\xi)\exp(-\mathrm{i}\Omega_0\tau)$, where Ω_0 is a constant. Hence, we readily obtain from equations (14) and (13)

$$2\mathrm{i}\omega\left[(-\mathbf{V}+\mathbf{v}_g)\cdot\nabla_\xi\right]\mathbf{a} + c^2\nabla^2_{\mathbf{r}}\mathbf{a} + \left(\Omega^2 - \omega_p^2 + 2\omega\Omega_0\right)\mathbf{a} - \omega_p^2\left[(N-\gamma_{e1}) - N\gamma_{e1}\right]\mathbf{a} = 0, \quad (15)$$

and

$$(\mathbf{V}\cdot\nabla_\xi)^2 N - C_s^2\,\nabla_\xi^2 N = \frac{m_0}{m_i}c^2\,\nabla_\xi^2\gamma_{e1}. \quad (16)$$

For the sake of simplicity, we consider here the one-dimensional case for which can write $(\mathbf{V}\cdot\nabla_\xi)^2 N = V^2\partial_\xi^2 N$. Consequently, equation (16) can be immediately integrated, yielding

$$N = \frac{m_0 c^2}{m_i\left(V^2 - C_s^2\right)}\,\gamma_{e1}. \quad (17)$$

Accounting for the explicit expression of γ_{e1}, choosing $|\mathbf{V}| = |\mathbf{v}_g| \gg C_s$, and combining equations (15, 17), we easily obtain

$$\frac{1}{2}\partial_\eta^2\boldsymbol{\Psi} + \lambda^2\boldsymbol{\Psi} - \left[q_1|\boldsymbol{\Psi}|^2 + q_2|\boldsymbol{\Psi}|^4\right]\boldsymbol{\Psi} = 0, \quad (18)$$

where we have introduced the following dimensionless quantities

$$\eta = c\,\xi/\omega_p,$$

$$\Psi = \frac{e\,\mathbf{a}}{\sqrt{2}m_0c^2},$$

$$\lambda^2 = \frac{\omega^2 - c^2k^2 - \omega_p^2 + 2\omega\Omega_0}{2\omega_p^2},$$

$$q_1 = \frac{1}{2}(\mu - 1)$$

and

$$q_2 = -\frac{1}{2}\mu,$$

with $\mu = m_0c^2/m_i(V^2 - C_s^2) \approx m_0c^2/m_iV^2$.

Note that, since $\mu > 0$, q_2 is a negative quantity. Additionally, since N and γ_{e1} must be of the same order, from equation (17) is evident that μ is of the order of the unity ($\mu \sim 1$), but slightly greater than 1. This circumstance is satisfied when we choose, consistently, a group velocity $v_g = V \sim (m_0/m_i)^{1/2}c$. This justifies why we kept both the nonlinear terms in equation (18); accordingly, the terms $q_1|\boldsymbol{\Psi}|^2$ and $q_2|\boldsymbol{\Psi}|^4$ are of the same order. In particular, if μ is exactly equal to 1 (*i.e.*, we have exactly $V = (m_0/m_i)^{1/2}c$), equation (18) becomes

$$\frac{1}{2}\partial_\eta^2\Psi + \lambda^2\boldsymbol{\Psi} - q_2|\boldsymbol{\Psi}|^4\boldsymbol{\Psi} = 0, \quad (19)$$

which shows that a part of the relativistic mass variation nonlinearity exactly cancelled out by the light ponderomotive force driven supersonic electron density contribution. If we put

$$\boldsymbol{\Phi}(\eta, s) = \Psi(\eta)\exp\left(-\mathrm{i}\lambda^2 s\right), \quad (20)$$

where s is a new dimensionless time-like variable, equation (18) can be cast as

$$i\partial_s\Phi + \frac{1}{2}\partial_\eta^2\boldsymbol{\Phi} - \left[q_1|\boldsymbol{\Phi}|^2 + q_2|\boldsymbol{\Phi}|^4\right]\boldsymbol{\Phi} = 0. \quad (21)$$

Let us suppose that μ is (slightly) larger than 1. In this way $q_1 > 0$, and equation (21) admits bright, gray and dark envelope soliton-like solutions. In fact, from the results of recent investigations [7] that have found a wide class of envelope soliton-like solutions of equation (21), one can deduce, through equation (20), the following soliton-like solution for $\Psi(\eta)$

$$\Psi(\eta) = \sqrt{\overline{u}\left[1 + \epsilon\,\mathrm{sech}\,(\eta/\Delta)\right]}\exp\left\{\mathrm{i}B\left[\frac{\eta}{\Delta} + \frac{2\epsilon}{\sqrt{1-\epsilon^2}} \times \arctan\left(\frac{(\epsilon - 1)\tanh(\eta/2\Delta)}{\sqrt{1-\epsilon^2}}\right)\right] + \mathrm{i}\phi_0\right\}, \quad (22)$$

where ϕ_0 is an arbitrary real constant, $\overline{u} = -3q_1/(8q_2) = 3(\mu-1)/(8\mu)$, the constants ϵ, Δ and B are given, respectively, by

$$\epsilon = \pm\sqrt{1 - \frac{32|q_2|V_0^2}{3q_1^2}} = \pm\sqrt{1 - \frac{64|\mu|V_0^2}{3\,(\mu-1)^2}},$$

$$\Delta = \left(2\sqrt{2\left|-\frac{3q_1^2}{64\,|q_2|} + \frac{V_0^2}{2}\right|}\right)^{-1}$$

$$= \left(2\sqrt{2\left|-\frac{3(\mu-1)^2}{128\,|\mu|} + \frac{V_0^2}{2}\right|}\right)^{-1},$$

$$B = V_0\Delta,$$

provided that

$$\lambda^2 = \frac{15(\mu-1)^2}{128|\mu|} + \frac{V_0^2}{2},$$

and the real constant V_0 satisfies the condition

$$-\sqrt{\frac{3(\mu-1)^2}{64\,|\mu|}} < V_0 < \sqrt{\frac{3(\mu-1)^2}{64\,|\mu|}}.$$

Accordingly, the definition of λ^2 implies that

$$\omega^2 - c^2k^2 + 2\omega\Omega_0 = \omega_p^2\left(1 + \frac{15(\mu-1)^2}{64\mu} + V_0^2\right),$$

which is a condition for the real constant Ω_0. According to the terminology and the results of references [7,8], we can distinguish the following four cases. (a) $0 < \epsilon < 1$ ($V_0 \neq 0$): *up-shifted bright soliton*

$$u(\eta = 0) = \overline{u}(1+\epsilon), \quad \text{and} \quad \lim_{\eta\to\pm\infty} u(\eta) = \overline{u}$$

which corresponds to a bright soliton of maximum amplitude $(1+\epsilon)\overline{u}$ and up-shifted by the quantity $\overline{u}$. (b) $-1 < \epsilon < 0$ ($V_0 \neq 0$): *gray soliton*

$$u(\eta = 0) = \overline{u}(1-\epsilon), \quad \text{and} \quad \lim_{\eta\to\pm\infty} u(\eta) = \overline{u}$$

which is a dark soliton with minimum amplitude $(1-\epsilon)\overline{u}$ and reaching the asymptotic upper limit $\overline{u}$. (c) $\epsilon = 1$ ($V_0 = 0$): *upper-shifted bright soliton*

$$u(\eta = 0) = 2\overline{u}, \quad \text{and} \quad \lim_{\eta\to\pm\infty} u(\eta) = \overline{u}$$

which corresponds to a bright soliton of maximum amplitude $2\overline{u}$ and *up*-shifted by the maximum quantity $\overline{u}$. (d) $\epsilon = -1$ ($V_0 = 0$): *standard dark soliton*

$$u(\eta = 0) = 0, \quad \text{and} \quad \lim_{\eta\to\pm\infty} u(\eta) = \overline{u}$$

which is a dark soliton (zero minimum amplitude), reaching the asymptotic upper limit $\overline{u}$.

Correspondingly, equation (17) gives the following soliton-like solution for the density fluctuation

$$N(\eta) = \mu\,|\Psi(\eta)|^2 = \frac{3}{8}(\mu-1)\left[1 + \epsilon\,\mathrm{sech}\,(\eta/\Delta)\right]. \quad (23)$$

On the other hand, according to references [7,8], equation (19) has the following bright envelope soliton-like solution

$$\Psi(\eta) = \left[\frac{6|E_0|}{\mu}\right]^{1/4} \mathrm{sech}^{1/2}\left[\sqrt{2|E_0|}\eta\right]\exp\left(\mathrm{i}\phi_0\right), \quad (24)$$

where ϕ_0 is an arbitrary real constant and E_0 is a negative real constant satisfying the condition $\lambda^2 = E_0$. The latter implies the following condition for Ω_0: $\omega^2 - c^2k^2 + 2\omega\Omega_0 + \omega_p^2\left(2|E_0| - 1\right) = 0$. Furthermore, equation (17) implies that now the density fluctuation corresponds to the following soliton-like solution

$$N(\eta) = \mu\,|\Psi(\eta)|^2 = \left[\frac{6|E_0|}{\mu}\right]^{1/2}\mathrm{sech}\left[\sqrt{2|E_0|}\eta\right]. \quad (25)$$

We now investigate the stability of plane wave solutions of the one-dimensional equation (21). We allow μ to run in a wider interval of values with respect to the one permitted in our physical problem; for the present analysis we allow μ to be also equal or less than 1 (including negative values). We first note that if we set $q_2 = 0$, the corresponding equation is the well known defocusing cubic Schrödinger equation which is known to be stable. It is, therefore, interesting to study if the $q_2|\Phi|^4$ term can modify the instability. The analysis is performed by seeking a solution corresponding to a uniform wave train perturbed by small disturbances, *viz.*

$$\Phi = [\Phi_0 + \rho(s,\eta)]\exp\mathrm{i}\{[-q_1|\Phi_0|^2 - q_2|\Phi_0|^4]s + \theta(s,\eta)\}, \quad (26)$$

where ρ and θ are considered to be small amplitude and small phase perturbations. We then substitute the perturbed solution in equation (21) and retain only the linear terms in ρ and θ. Since the resulting equation is linear we can now assume a solution for the perturbation of the form $\rho = \rho_0\exp\mathrm{i}[K\eta - \Omega s]$ and $\theta = \theta_0\exp\mathrm{i}[K\eta - \Omega s]$. The resulting dispersion relation is

$$\Omega^2 = \frac{K^2}{4}\left(K^2 + 4q_1|\Phi_0|^2 + 8q_2|\Phi_0|^4\right), \quad (27)$$

which shows that the wave train is unstable if the perturbation K lies in the range of $0 < K < 2|\Phi_0|\sqrt{-q_1 - 2q_2|\Phi_0|^2}$. According to the definition of q_1 and q_2, instability will occur only if $\mu > 1/(1 - 2|\Phi_0|^2)$. The maximum instability occurs at $K = |\Phi_0|\sqrt{-2q_1 - 4q_2|\Phi_0|^2}$.

4 Numerical simulations

In this section, we analyze numerically both the influence of the quintic nonlinearity on the modulational instability and on the stability of a class of soliton-like solutions obtained in the previous sections. Equation (21) is solved numerically using a standard pseudo-spectral code with a second order Runge-Kutta method for advancing in time. We recall that the use of pseudo-spectral code implies the assumption of periodic boundary conditions.

4.1 Modulational instability

Accordingly to the linear stability analysis performed previously, initial conditions for our numerical simulations are given as follows:

$$\Phi(x,0) = \Phi_0[1 + \varepsilon\cos(Lx)], \quad (28)$$

where ε is the amplitude of the small perturbation and is taken as 10^{-2} the amplitude of the unperturbed wave. Without loss of generality in our simulations we have chosen $\Phi_0 = 1$ and $L = 1$ and have considered only one period of the perturbation. We have performed several numerical simulations with different values of the parameter μ. For $\Phi_0 = 1$ the theory predicts stability for $\mu < -1$. In Figure 1 we show the evolution of a plane wave in the $\eta - s$ plane for $\mu = -1.5$. The initial wave field persists for all times. For $\mu > 1$ modulational instability should occur. In Figure 2 we show the case of $\mu = -0.2$; analogously with the standard modulational instability we observe a Fermi-Pasta-Ulam recurrence: periodically the perturbation grows, the wave reaches a maximum amplitude and

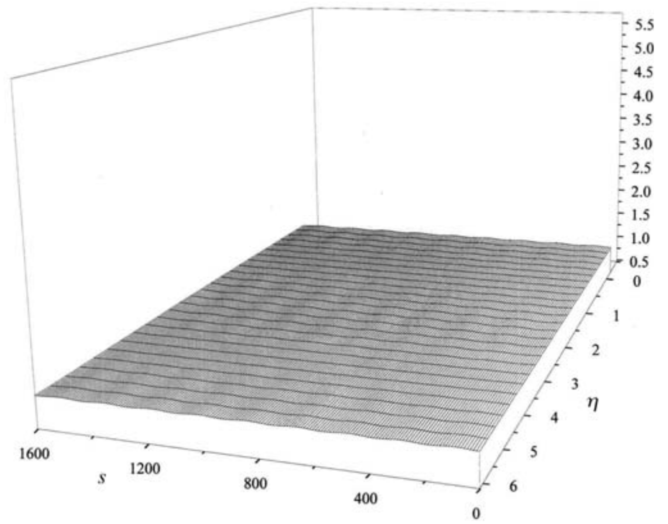

Fig. 1. Evolution of a plane wave in the η-s plane for $\mu = -1.5$. The initial wave field persists for all times.

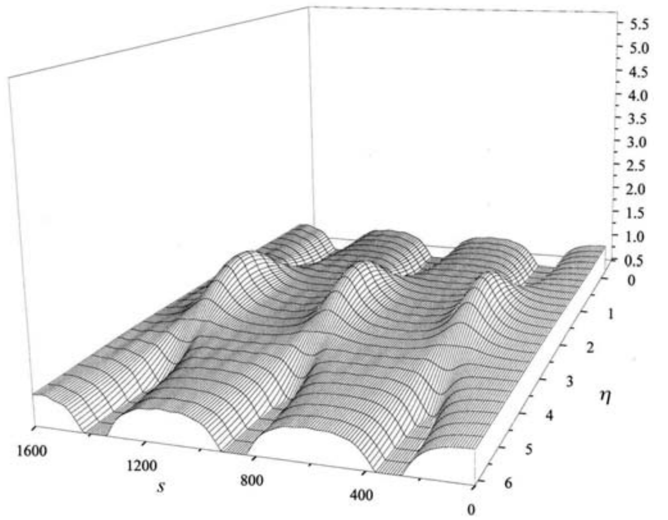

Fig. 2. Evolution of a plane wave in the η-s plane for $\mu = -0.2$; the wave reaches a maximum amplitude and then goes back to the initial condition.

then goes back to the initial condition. For $\mu > 0$ a completely different physics takes place: in Figure 3, obtained for $\mu = 0.5$, we do not observe anymore a recurrence and as time passes the wave amplitude increases while the its width decreases. This phenomenon corresponds to the initial stage of a wave collapse (see [21]).

4.2 Stability of soliton-like solutions

It is well know that the cubic NLS equations ($q_2 = 0$) posses solitons solutions if q_1 is larger than zero. We here investigate numerically if the soliton-like solutions described previously are stable or not. In order to do that we simply consider a soliton-like solution at time $s = 0$ and we let evolve numerically equation (21). For simplicity, we restrict our analysis to a sub class of solutions which corresponds to the case of bright solitons with $V_0 = 0$. We have performed many different numerical simulations with

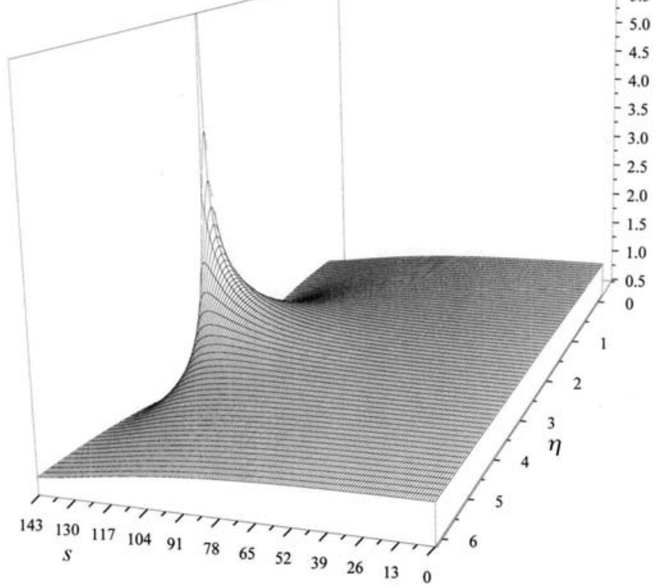

Fig. 3. Evolution of a plane wave in the η-s plane for $\mu = 0.5$. No recurrence is observed anymore and as time passes the wave amplitude increases while its width decreses.

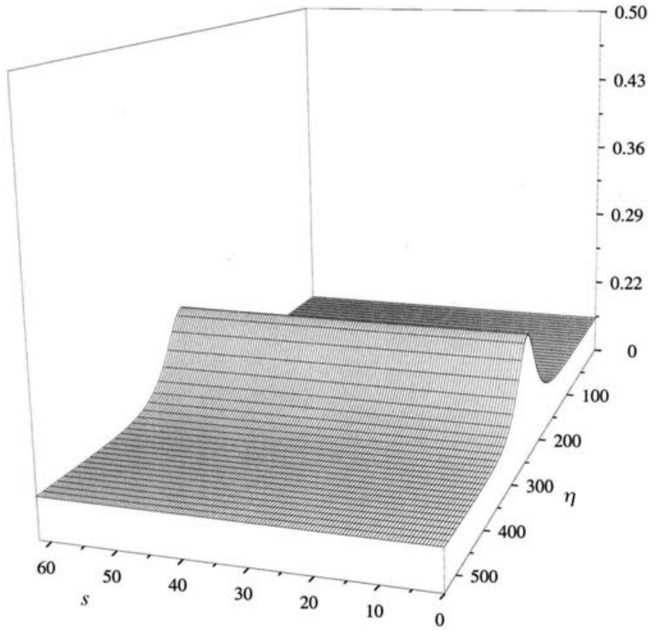

Fig. 4. Evolution of the soliton in the η-s plane for $\mu = 1.1$ which corresponds to a stable solution.

different values of the parameter μ. The major result obtained is the following: if $\mu > 1$ solutions are stable and for $0 < \mu < 1$ are unstable. This is due to the fact that for $\mu > 1$ the coefficients in front of the cubic and quintic nonlinearities, respectively q_1 and q_2, have opposite sign and, therefore, there is a sort of balance between nonlinearities that stabilize the soliton-like solution. This is not the case for $0 < \mu < 1$: both nonlinearities have the same sign and the dispersion is not strong enough to balance them. In Figures 4 and 5 we give numerical evidence of the results presented for $\mu = 1.1$ and for $\mu = 0.9$ respectively. The first case, Figure 4, corresponds to a stable solutions (the

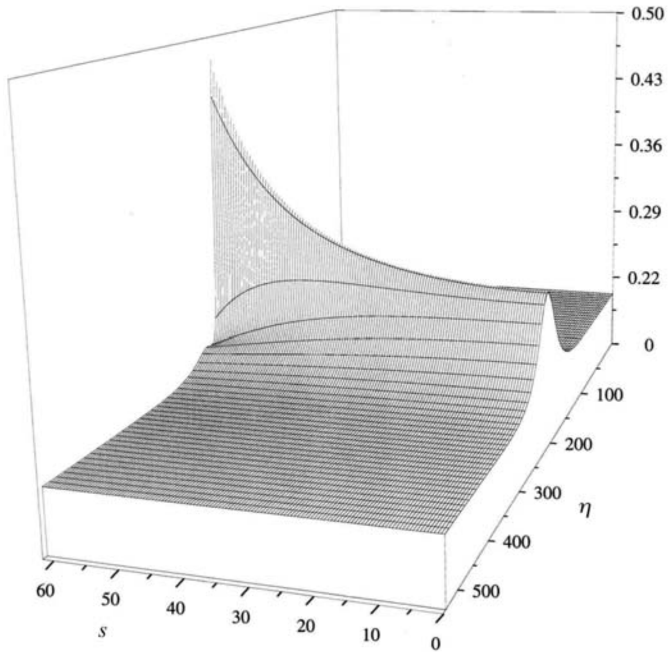

Fig. 5. Evolution of the soliton in the η-s plane for $\mu = 0.9$ which corresponds to an unstable solution.

wave profile does not change as time s passes). The second case, Figure 5, is the unstable case: a clear increase in the wave amplitude is noted.

According to the above stability analysis, we conclude that our soliton solutions are stable in the range where $\mu > 1$ only; this inequality, according to Section 3, is consistent with the conditions for their existence in our problem.

5 Conclusions

To summarize, we have considered the nonlinear interaction between intense light beams and non-resonant density perturbations, taking into account the relativistic mass increase of the electrons as well as the light beam ponderomotive force that reinforces the density perturbations in an electron-ion plasma. The nonlinear coupling is governed by a pair of equations which, in one-dimension admit stationary solutions in the form of a planar bright and dark/gray envelope solitons. The condition for the stability of bright soliton-like solutions has been found numerically and it has shown that they are stable just for the range of parameters required in our problem.

The numerical stability analysis for the family of dark and gray solitons, which is somewhat more complex, is under consideration.

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